

# Markscheme

November 2019

Mathematical studies

Standard level

Paper 2

28 pages

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**Paper 2 Markscheme  
Instructions to Examiners**

**Notes:** If in doubt about these instructions or any other marking issues, contact your team leader for clarification.

**1 Abbreviations**

- M** Marks awarded for **Method**
- A** Marks awarded for an **Answer** or for **Accuracy**
- R** Marks awarded for clear **Reasoning**
- G** Marks awarded for correct solutions obtained from a **Graphic Display Calculator**, when no working shown.
- AG** **Answer Given** in the question and consequently, marks not awarded.
- ft** Marks that can be awarded as **follow through** from previous results in the question.

**2 Method of Marking**

- (a) All marking must be done in RM Assessor using the mathematical studies annotations and in accordance with the current document for guidance in e-marking Mathematical Studies SL. It is essential that you read this document before you start marking.
- (b) If a question part is completely correct use the number tick annotations to award full marks. If a part is completely wrong use the **A0** annotation, otherwise full annotations must be shown.
- (c) Working crossed out by the candidate should not be awarded any marks.
- (d) Where candidates have written two solutions to a question, only the first solution should be marked.
- (e) If correct working results in a correct answer but then further working is developed, indicating a lack of mathematical understanding full marks should **not** be awarded. In most such cases it will be a single final answer mark that is lost. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal.

**Example:**

	Correct answer seen	Further working seen	Action
1.	$8\sqrt{2}$	5.65685... (incorrect decimal value)	Award the final <b>(A1)</b> (ignore the further working)
2.	$(x-6)(x+1)$	$x=6$ and $-1$	Do <b>not</b> award the final <b>(A1)</b>

**Example:** Calculate the gradient of the line passing through the points (5, 3) and (0, 9) .

Markscheme	Candidates' Scripts	Marking
$\frac{9-3}{0-5}$ <p><b>(M1)</b></p> <p>Award <b>(M1)</b> for correct substitution in gradient formula</p> $= -\frac{6}{5}$ <p><b>(A1)</b></p>	(i) $\frac{9-3}{0-5} = -\frac{6}{5}$ <p><b>(M1)</b></p> <p>Gradient is <math>= -\frac{6}{5}</math></p> <p><b>(A1)</b></p> <p>(There is clear understanding of the gradient.)</p> $y = -\frac{6}{5}x + 9$	
	(ii) $\frac{9-3}{0-5} = -\frac{6}{5}$ <p><b>(M1)</b></p> $y = -\frac{6}{5}x + 9$ <p><b>(A0)</b></p> <p>(There is confusion about what is required.)</p>	

### 3 Follow-through (ft) Marks

Errors made at any step of a solution affect all working that follows. To limit the severity of the penalty, **follow through (ft)** marks can be awarded. Mark schemes will indicate where it is appropriate to apply follow through in a question with **'(ft)'**.

- (a) Follow through applies only from one part of a question to a subsequent part of the question. Follow through does not apply within the same part.
- (b) If an answer resulting from follow through is extremely unrealistic (eg, negative distances or incorrect by large order of magnitude) then the final **A** mark should not be awarded.
- (c) If a question is transformed by an error into a **different, much simpler question** then follow through may not apply.
- (d) To award follow through marks for a question part, **there must be working present for that part**. An isolated follow through answer, without working is regarded as incorrect and receives no marks **even if it is approximately correct**.
- (e) The exception to the above would be in a question which is testing the candidate's use of the GDC, where working will not be expected. **The markscheme will clearly indicate where this applies**.
- (f) Inadvertent use of radians will be penalized the first time it occurs. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for the use of radians.

**Example:** Finding angles and lengths using trigonometry

Markscheme	Candidates' Scripts	Marking
<p>(a) <math>\frac{\sin A}{3} = \frac{\sin 30}{4}</math> <b>(M1)(A1)</b>                      Award <b>(M1)</b> for substitution in sine rule formula, <b>(A1)</b> for correct substitutions.  <math>A = 22.0^\circ</math> (22.0243...) <b>(A1)(G2)</b></p>	<p>(a) <math>\frac{\sin A}{4} = \frac{\sin 30}{3}</math>  <math>A = 41.8^\circ</math>                      (<b>Note:</b> the 2<sup>nd</sup> <b>(A1)</b> here was not marked <b>(ft)</b> and cannot be awarded because there was an earlier error in the <b>same</b> question part.)</p>	<p><b>(M1)(A0)</b>                      (use of sine rule but with wrong values)  <b>(A0)</b></p>
<p>(b) <math>x = 7 \tan (22.0243\dots^\circ)</math> <b>(M1)</b>  <math>= 2.83</math> (2.83163...) <b>(A1)(ft)</b></p>	<p>(b) case (i) <math>x = 7 \tan 41.8^\circ</math>  <math>= 6.26</math>  <b>but</b> case (ii) 6.26</p>	<p><b>(M1)</b>  <b>(A1)(ft)</b>  <b>(G0)</b>                      since no working shown</p>

**4 Using the Markscheme**

- (a) **A** marks are **dependent** on the preceding **M** mark being awarded, it is **not** possible to award **(M0)(A1)**. Once an **(M0)** has been awarded, all subsequent **A** marks are lost in that part of the question, even if calculations are performed correctly, until the next **M** mark. The only exception to this will be for an answer where the accuracy is specified in the question – see section 5.
- (b) **A** marks are **dependent** on the **R** mark being awarded, it is **not** possible to award **(A1)(R0)**. Hence the **(A1)** cannot be awarded for an answer which is correct when no reason or the wrong reason is given.
- (c) In paper 2 candidates are expected to demonstrate their ability to communicate mathematics using appropriate working. Answers which are correct but not supported by adequate working will **not always receive full marks**, these unsupported answers are designated **G** in the mark scheme as an alternative to the full marks. Example **(M1)(A1)(A1)(G2)**.

**Example:** Using trigonometry to calculate an angle in a triangle.

Markscheme	Candidates' Scripts	Marking
(a) $\frac{\sin A}{3} = \frac{\sin 30}{4}$ <b>(M1)(A1)</b> Award <b>(M1)</b> for substitution in sine rule formula, <b>(A1)</b> for correct substitutions.  $A = 22.0^\circ$ (22.0243...) <b>(A1)(G2)</b>	(i) $\frac{\sin A}{3} = \frac{\sin 30}{4}$ $A = 22.0^\circ$  (ii) $A = 22.0^\circ$	<b>(M1)(A1)</b>  <b>(A1)</b>  <b>(G2)</b> <b>Note: G marks are used only if no working has been shown and the answer is correct.</b>

- (d) **Alternative methods** may not always be included. Thus, if an answer is wrong then the working must be carefully analysed in order that marks are awarded for a different method consistent with the markscheme. Where alternative methods for complete questions are included in the markscheme, they are indicated by '**OR**' etc.
- (e) Unless the question specifies otherwise, accept **equivalent forms**. For example:  $\frac{\sin \theta}{\cos \theta}$  for  $\tan \theta$ .  
 On the markscheme, these equivalent numerical or algebraic forms will sometimes be written in brackets after the required answer.  
 Where numerical answers are required as the final answer to a part of a question in the markscheme, the scheme will show, in order:  
 the 3 significant figure answer worked through from full calculator display;  
 the exact value  $\left( \text{for example } \frac{2}{3} \text{ if applicable} \right)$ ;  
 the full calculator display in the form 2.83163... as in the example above.  
 Where answers are given to 3 significant figures and are then used in subsequent parts of the question leading to a **different** 3 significant figure answer, these solutions will also be given.

- (f) As this is an international examination, all valid **alternative forms of notation** should be accepted. Some examples of these are:

Decimal points: 1.7; 1'7; 1·7; 1,7 .

Decimal numbers less than 1 may be written with or without a leading zero: 0.49 or .49 .

Different descriptions of an interval:  $3 < x < 5$ ;  $(3, 5)$ ;  $] 3, 5 [$  .

Different forms of notation for set properties (e.g. complement):  $A'$ ;  $\bar{A}$ ;  $A^c$ ;  $U - A$ ;  $(A; U \setminus A$ .

Different forms of logic notation:  $\neg p$ ;  $p'$ ;  $\tilde{p}$ ;  $\bar{p}$ ;  $\sim p$ ;  $-p$ .  
 $p \Rightarrow q$ ;  $p \rightarrow q$ ;  $q \Leftarrow p$  .

Significance level may be written as  $\alpha$  .

- (g) Discretionary marks: There will be very rare occasions where the markscheme does not cover the work seen. In such cases the annotation DM should be used to indicate where an examiner has used discretion. Discretion should be used sparingly and if there is doubt an exception should be raised through RM Assessor to the team leader.

As with previous sessions there will be no whole paper penalty marks for accuracy AP, financial accuracy FP and units UP. Instead these skills will be assessed in particular questions and the marks applied according to the rules given in sections 5, 6 and 7 below.

### 5 Accuracy of Answers

Incorrect accuracy should be penalized once only in each question according to the rules below.

Unless otherwise stated in the question, all numerical answers should be given exactly or correct to 3 significant figures.

1. If the **candidate's answer** is seen to 4 sf or greater **and** would round to the required 3 sf answer, then award **(A1)** and ignore subsequent rounding.
2. If the candidate's unrounded answer is **not** seen then award **(A1)** if the answer given is **correctly** rounded to 2 or more significant figures, otherwise **(A0)**.

**Note:** If the candidate's unrounded answer is **not** seen and the answer is given correct to 1 sf (correct or not), the answer will be considered wrong and will not count as incorrect accuracy. If this answer is used in subsequent parts, then working must be shown for further marks to be awarded.

3. If a correct 2 sf answer is used in subsequent parts, then working **must** be shown for further marks to be awarded. (This treatment is the same as for following through from an incorrect answer.)

These 3 points (see numbers in superscript) have been summarized in the table below and illustrated in the examples following.

	If candidates final answer is given ...					
	Exact or to 4 or more sf (and would <b>round to the correct 3 sf</b> )	<b>Correct to 3 sf</b>	<b>Incorrect to 3 sf</b>	Correct to 2 sf <sup>3</sup>	Incorrect to 2 sf	Correct or incorrect to 1 sf
Unrounded answer seen <sup>1</sup>	Award the final <b>(A1)</b> irrespective of correct or incorrect rounding					
Unrounded answer not seen <sup>2</sup>	<b>(A1)</b>	<b>(A1)</b>	<b>(A0)</b>	<b>(A1)</b>	<b>(A0)</b>	<b>(A0)</b>
Treatment of subsequent parts	As per MS		Treat as follow through, only if working is seen. <sup>3</sup>			

Examples:

Markscheme	Candidates' Scripts	Marking
9.43 (9.43398...) <b>(A1)</b>	(i) 9.43398... is seen followed by 9; 9.4; 9.43; 9.434 etc. (correctly rounded)	<b>(A1)</b>
	(ii) 9.43398... is seen followed by 9.433; 9.44 etc. (incorrectly rounded)	<b>(A1)</b>
	(iii) 9.4	<b>(A1)</b>
	(iv) 9	<b>(A0)</b> (correct to 1 sf)
	(v) 9.3	<b>(A0)</b> (incorrectly rounded to 2 sf)
	(vi) 9.44	<b>(A0)</b> (incorrectly rounded to 3 sf)

Markscheme	Candidates' Scripts	Marking
7.44 (7.43798...) <b>(A1)</b>	(i) 7.43798... is seen followed by 7; 7.4; 7.44; 7.438 etc. (correctly rounded)	<b>(A1)</b>
	(ii) 7.43798... is seen followed by 7.437; 7.43 etc. (incorrectly rounded)	<b>(A1)</b>
	(iii) 7.4	<b>(A1)</b>
	(iv) 7	<b>(A0)</b> (correct to 1 sf)
	(v) 7.5	<b>(A0)</b> (incorrectly rounded to 2 sf)
	(vi) 7.43	<b>(A0)</b> (incorrectly rounded to 3 sf)



**Example:** ABC is a right angled triangle with angle  $ABC = 90^\circ$ ,  $AC = 32$  cm and  $AB = 30$  cm. Find (a) the length of BC, (b) The area of triangle ABC.

Markscheme	Candidates' Scripts	Marking
(a) $BC = \sqrt{32^2 - 30^2}$ <b>(M1)</b> Award <b>(M1)</b> for correct substitution in Pythagoras' formula  $= 11.1 (\sqrt{124}, 11.1355\dots)$ (cm) <b>(A1)</b>	(a) $BC = \sqrt{32^2 - 30^2}$  11 (cm)  <i>(2 sf answer only seen, but correct)</i>	<b>(M1)</b>  <b>(A1)</b>
(b) $\text{Area} = \frac{1}{2} \times 30 \times 11.1355\dots$ <b>(M1)</b> Award <b>(M1)</b> for correct substitution in area of triangle formula  $= 167(167.032\dots)$ (cm <sup>2</sup> ) <b>(A1)(ft)</b>	(b) case (i) $\text{Area} = \frac{1}{2} \times 30 \times 11$  $= 165$ (cm <sup>2</sup> )  case (ii) $= 165$ (cm <sup>2</sup> )  <i>(No working shown, the answer 11 is treated as a ft, so no marks awarded here)</i>	<b>(M1)</b> <i>(working shown)</i>  <b>(A1)(ft)</b>  <b>(M0)(A0)(ft)</b>

Certain answers obtained from the GDC are worth 2 marks and working will not be seen. In these cases only one mark should be lost for accuracy.

eg, Chi-squared, correlation coefficient, mean

Markscheme	Candidates' Scripts	Marking
Chi-squared	(a) 7.7	<b>(G2)</b>
7.68 (7.67543...) <b>(A2)</b>	(b) 7.67	<b>(G1)</b>
	(c) 7.6	<b>(G1)</b>
	(d) 8	<b>(G0)</b>
	(e) 7	<b>(G0)</b>
	(e) 7.66	<b>(G0)</b>

Regression line

Markscheme	Candidates' Scripts	Marking
$y = 0.888x + 13.5$ <b>(A2)</b> ( $y = 0.887686\dots x + 13.4895\dots$ ) If an answer is not in the form of an equation award at most <b>(A1)(A0)</b> .	(a) $y = 0.89x + 13$	<b>(G2)</b> (both accepted)
	(b) $y = 0.88x + 13$	<b>(G1)</b> (one rounding error)
	(c) $y = 0.88x + 14$	<b>(G1)</b> (rounding error repeated)
	(d) (i) $y = 0.9x + 13$	<b>(G1)</b> (1 sf not accepted)
	(ii) $y = 0.8x + 13$	
(e) $0.88x + 13$	<b>(G0)</b> (one rounding error and not an equation)	

Maximum/minimum/points of intersection

Markscheme	Candidates' Scripts	Marking
(2.06, 4.49) <b>(A1)(A1)</b> (2.06020..., 4.49253...)	(a) (2.1, 4.5)	<b>(A1)(A1)</b> (both accepted)
	(b) (2.0, 4.4)	<b>(A1)</b> (same rounding error twice)
	(c) (2.06, 4.4)	<b>(A1)</b> (one rounding error)
	(d) (2, 4.4)	<b>(A0)</b> (1sf not accepted, one rounding error)

Rounding of an exact answer to 3 significant figures **should be accepted if performed correctly**.

Exact answers such as  $\frac{1}{4}$  can be written as decimals to fewer than 3 significant figures if the result is still exact. Reduction of a fraction to its lowest terms is **not** essential, however where an answer simplifies to an integer this is expected. Fractions that include a decimal in the numerator and/or the denominator are acceptable for showing correct substitution, but not as a final answer.

Ratios of  $\pi$  and answers taking the form of square roots of integers or any rational power of an integer (eg,  $\sqrt{13}, 2^{\frac{2}{3}}, \sqrt[4]{5}$ , ) may be accepted as exact answers. All other powers (eg, of non-integers) and values of transcendental functions such as sine and cosine must be evaluated.

**If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy.** In all such cases the final mark is not awarded if the rounding does not follow the instructions given in the question. A mark for specified accuracy can be regarded as a **(ft)** mark regardless of an immediately preceding **(M0)**.

**6 Level of accuracy in finance questions**

The accuracy level required for answers will be specified in all questions involving money. This will usually be either whole units or two decimal places. The first answer not given to the specified level of accuracy will not be awarded the final **A** mark. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for incorrect accuracy in a financial question.

**Example:** A financial question demands accuracy correct to 2 dp.

Markscheme	Candidates' Scripts	Marking
\$231.62 (231.6189) <b>(A1)</b>	(i) 231.6	<b>(A0)</b>
	(ii) 232	<b>(A0)</b> <i>(Correct rounding to incorrect level)</i>
	(iii) 231.61	<b>(A0)</b>
	(iv) 232.00	<b>(A0)</b> <i>(Parts (iii) and (iv) are both incorrect rounding to correct level)</i>

**7 Units in answers**

There will be specific questions for which the units are required and this will be indicated clearly in the markscheme. The first correct answer with no units or incorrect units will not be awarded the final **A** mark. The markscheme will give clear instructions to ensure that only one or two mark per paper can be lost for lack of units or incorrect units.

The units are considered only when the numerical answer is awarded **(A1)** under the accuracy rules given in Section 5.

**Example:**

Markscheme	Candidates' Scripts	Marking
(a) 37000 m <sup>2</sup> <b>(A1)</b>	(a) 36000 m <sup>2</sup>	<b>(A0)</b> <i>(Incorrect answer so units not considered)</i>
(b) 3200 m <sup>3</sup> <b>(A1)</b>	(b) 3200 m <sup>2</sup>	<b>(A0)</b> <i>(Incorrect units)</i>

**If no method is shown and the answer is correct but with incorrect or missing units award G marks with a one mark penalty.**

**8 Graphic Display Calculators**

Candidates will often be obtaining solutions directly from their calculators. They must use mathematical notation, not calculator notation. No method marks can be awarded for incorrect answers supported only by calculator notation. The comment 'I used my GDC' cannot receive a method mark.

1. (a) ( $H_0$  : ) choice of meal is independent of age (or equivalent) (A1)

**Note:** Accept “not associated” or “not dependent” instead of independent. In lieu of “age”, accept an equivalent alternative such as “being a child or adult”.

[1 mark]

- (b) 2 (A1)

[1 mark]

- (c)  $\frac{69}{150} \times \frac{67}{150} \times 150$  OR  $\frac{69 \times 67}{150}$  (M1)

**Note:** Award (M1) for correct substitution into expected frequency formula.

30.82 (30.8) (A1)

31 (AG)

**Note:** Both an unrounded answer that rounds to the given answer and rounded answer must be seen for the (A1) to be awarded.

[2 marks]

- (d) (i) ( $\chi^2_{\text{calc}} =$ ) 2.66 (2.657537...) (G2)

- (ii) ( $p$ -value =) 0.265 (0.264803...) (G1)

**Note:** Award (G0)(G2) if the  $\chi^2$  statistic is missing or incorrect and the  $p$ -value is correct.

[3 marks]

- (e)  $0.265 > 0.10$  OR  $2.66 < 4.605$  (R1)(ft)

the null hypothesis is not rejected (A1)(ft)

**OR**

the choice of meal is independent of age (or equivalent) (A1)(ft)

**Note:** Award (R1)(ft) for a correct comparison of either their  $\chi^2$  statistic to the  $\chi^2$  critical value or their  $p$ -value to the significance level. Condone “accept” in place of “not reject”. Follow through from parts (a) and (d).

Do not award (A1)(ft)(R0).

[2 marks]

continued...

Question 1 continued

(f) (i)  $\frac{81}{150} \left( \frac{27}{50}, 0.54, 54\% \right)$  (A1)(A1)(G2)

**Note:** Award (A1) for numerator, (A1) for denominator.

(ii)  $\frac{116}{150} \left( \frac{58}{75}, 0.773, 0.773333\dots, 77.3\% \right)$  (A1)(A1)(G2)

**Note:** Award (A1) for numerator, (A1) for denominator.

(iii)  $\frac{34}{69} (0.493, 0.492753\dots, 49.3\%)$  (A1)(A1)(G2)

**Note:** Award (A1) for numerator, (A1) for denominator.

[6 marks]

Total [15 marks]

2. (a)  $\frac{2 - (-1)}{-3 - (-9)}$  (M1)

**Note:** Award (M1) for correct substitution into the gradient formula.

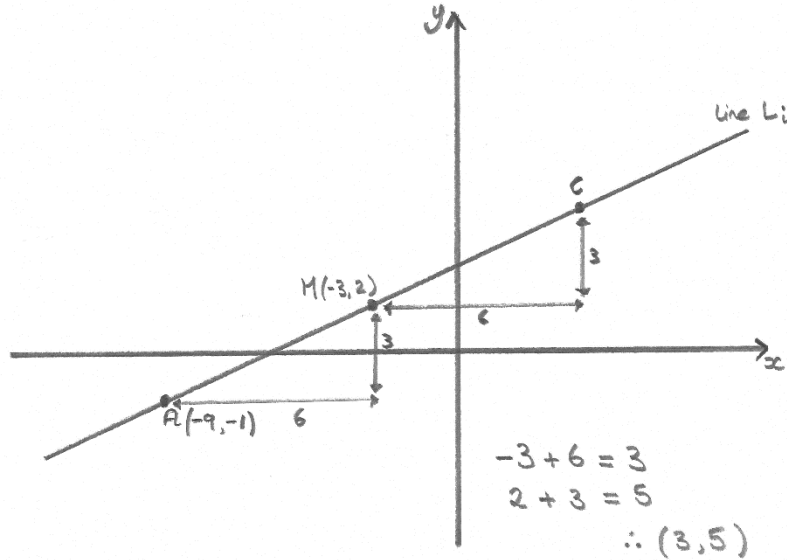
$= \frac{1}{2} \left( \frac{3}{6}, 0.5 \right)$  (A1)(G2)

[2 marks]

(b)  $-3 = \frac{-9+x}{2} (-6+9=x)$  and  $2 = \frac{-1+y}{2} (4+1=y)$  (M1)

**Note:** Award (M1) for correct substitution into the midpoint formula for both coordinates.

OR



(M1)

**Note:** Award (M1) for a sketch showing the horizontal displacement from M to C is 6 and the vertical displacement is 3 and the coordinates at M.

OR

$-3 + 6 = 3$  and  $2 + 3 = 5$  (M1)

**Note:** Award (M1) for correct equations seen.

(3, 5) (A1)(G1)(G1)

**Note:** Accept  $x = 3, y = 5$ . Award at most (M1)(A0) or (G1)(G0) if parentheses are missing.

[2 marks]

continued...

Question 2 continued

(c) gradient of the normal = -2 (A1)(ft)

**Note:** Follow through from their gradient from part (a).

$$y - 2 = -2(x + 3) \quad \text{OR} \quad 2 = -2(-3) + c \quad \text{(M1)}$$

**Note:** Award (M1) for correct substitution of M and their gradient of normal into straight line formula.

$$2x + y + 4 = 0 \quad (\text{accept integer multiples}) \quad \text{(A1)(ft)(G3)}$$

[3 marks]

(d)  $2(k) + 4 + 4 = 0$  (M1)

**Note:** Award (M1) for substitution of  $y = 4$  into their equation of normal line or substitution of M and  $(k, 4)$  into equation of gradient of normal.

$$k = -4 \quad \text{(A1)(ft)(G2)}$$

**Note:** Follow through from part (c).

[2 marks]

(e)  $\sqrt{(-4 + 3)^2 + (4 - 2)^2}$  (M1)

**Note:** Award (M1) for correctly substituting point M and their N into distance formula.

$$\sqrt{5} \quad (2.24, 2.23606\dots) \quad \text{(A1)(ft)}$$

**Note:** Follow through from part (d).

[2 marks]

(f)  $\frac{1}{2} \times (2 \times \sqrt{45}) \times \sqrt{5}$  (M1)

**Note:** Award (M1) for their correct substitution into area of a triangle formula.

Award (M0) for *their*  $\frac{1}{2} \times (\sqrt{45}) \times \sqrt{5}$  without any evidence of multiplication by 2 to find length AC. Accept any other correct method to find the area.

$$15 \quad \text{(A1)(ft)(G2)}$$

**Note:** Accept 15.02637... from use of a 3 sf value for  $\sqrt{5}$ .  
Follow through from part (e).

[2 marks]

**Total [13 marks]**

3. (a)  $u_{12} = 5 + (12 - 1) \times (2)$ . (M1)(A1)

**Note:** Award (M1) for substituted arithmetic sequence formula, (A1) for correct substitutions.

$27$  (A1)(G3)

[3 marks]

(b)  $S_n = \frac{n}{2}(2 \times 5 + (n - 1)(2))$  (M1)(A1)

**Note:** Award (M1) for substituted arithmetic series formula, (A1) for correct substitutions.

$S_n = \frac{n}{2}(8 + 2n)$  OR  $S_n = n(5 + n - 1)$  (M1)

**Note:** Award (M1) for evidence of expansion and simplification, or division by 2 leading to the final answer.

$S_n = n^2 + 4n$  (AG)

**Note:** The final line must be seen, with no incorrect working, for the final (M1) to be awarded.

[3 marks]

(c)  $(S_{18} =) 18^2 + 4 \times 18$  (M1)

**Note:** Award (M1) for correctly substituted formula for  $S_n$ .

$(S_{18} =) 396$  (A1)

**Note:** The use of “hence” in the question paper means that the  $S_n$  formula (from part (b)) must be used.

[2 marks]

continued...



Question 3 continued

(d)  $1000 = n^2 + 4n$  OR  $1000 = \frac{n}{2}(10 + (n-1)2)$  (or equivalent) **(M1)**

**Note:** Award **(M1)** for equating  $S_n$  to 1000 or for equating the correctly substituted sum of arithmetic sequence formula to 1000.

**OR**

a sketch of the graphs  $S_n = n^2 + 4n$  and  $S_n = 1000$  intersecting **(M1)**

**Note:** Award **(M1)** for a sketch of a quadratic and a horizontal line with at least one point of intersection.

**OR**

a sketch of  $n^2 + 4n - 1000$  intersecting the  $x$ -axis **(M1)**

**Note:** Award **(M1)** for a sketch of  $n^2 + 4n - 1000$  with at least one  $x$ -intercept.

$(n =) 29.6859\dots$  OR  $-2 + 2\sqrt{251}$  **(A1)**

**Note:** Award **(A1)** for 29.6859... or  $-2 + 2\sqrt{251}$  seen. Can be implied by a correct final answer.

$(n =) 29$  **(A1)(ft)(G2)**

**Note:** Do not accept 30. Award a maximum of **(M1)(A1)(A0)** if two final answers are given. Follow though from their unrounded answer.

**OR**

$S_{30} = 1020$  and  $S_{29} = 957$  **(A2)**

**Note:** Award **(A2)** for both “crossover” values seen. Do not split this **(A2)** mark.

$(n =) 29$  **(A1)(G2)**

**[3 marks]**

continued...

Question 3 continued

(e)  $(A =) (29^2 + 4 \times 29) \times (1.84)$  **(M1)(M1)**

**Note:** Award **(M1)** for their correct substitution to find the total number of triangular panes. Award **(M1)** for multiplying their number of panes by 1.84 .

**OR**

$(A =) 957 \times 1.84$  **(A1)(ft)(M1)**

**Note:** Award **(A1)(ft)** for their 957 seen. Award **(M1)** for multiplying their number of panes by 1.84. Follow through from part (d).

$(A =) 1760.88(\text{m}^2)$  **(A1)(ft)(G2)**

$(A =) 1761(\text{m}^2)$  **(A1)(ft)(G3)**

**[4 marks]**

**Total [15 marks]**

4. (a)  $\frac{1}{2}(0-2)(0+8)$  OR  $\frac{1}{2}(0^2 + 6(0) - 16)$  (or equivalent) (M1)

**Note:** Award (M1) for evaluating  $f(0)$ .

$(c =) -8$  (A1)(G2)

**Note:** Award (G2) if  $-8$  or  $(0, -8)$  seen.

[2 marks]

(b)  $x = -3$  (A1)(A1)

**Note:** Award (A1) for " $x = \text{constant}$ ", (A1) for the constant being  $-3$ . The answer must be an equation.

[2 marks]

(c)  $(-3 - -10) + -3$  (M1)

OR

$(-8 - -10) + 2$  (M1)

OR

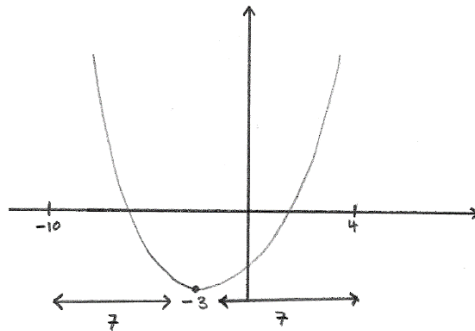
$\frac{-10+x}{2} = -3$  (M1)

OR

diagram showing axis of symmetry and given points ( $x$ -values labels,  $-10, -3$  and  $4$ , are sufficient) **and** an indication that the horizontal distances between the axis of symmetry and the given points are  $7$ . (M1)



OR



**Note:** Award (M1) for correct working using the symmetry between  $x = -10$  and  $x = -3$ . Award (M0) if candidate has used  $x = -10$  and  $x = 4$  to show the axis of symmetry is  $x = -3$ . Award (M0) if candidate solved  $f(x) = 12$  or evaluated  $f(-10)$  and  $f(4)$ .

$(x =) 4$  (AG)

[1 mark]

continued...

Question 4 continued

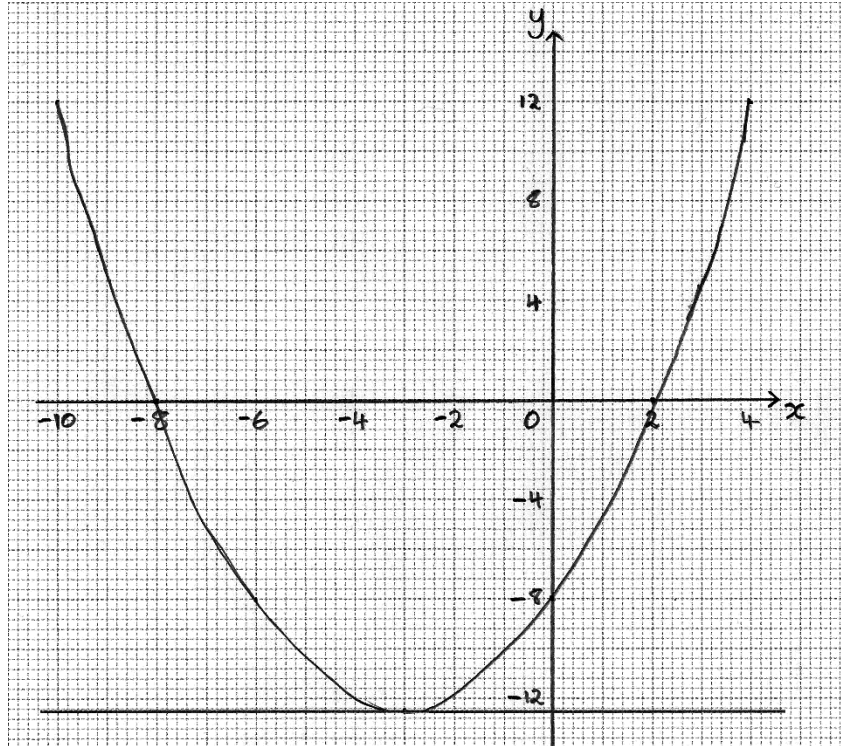
(d) -8 and 2

(A1)(A1)

**Note:** Accept  $x = -8, y = 0$  and  $x = 2, y = 0$  or  $(-8, 0)$  and  $(2, 0)$ , award at most (A0)(A1) if parentheses are omitted.

[2 marks]

(e)



(A1)(A1)(A1)(A1)(ft)

**Note:** Award (A1) for labelled axes with correct scale, correct window. Award (A1) for the vertex,  $(-3, -12.5)$ , in correct location.

Award (A1) for a smooth continuous curve symmetric about their vertex. Award (A1)(ft) for the curve passing through their  $x$  and  $y$  intercepts in correct location. Follow through from their parts (a) and (d).

**If graph paper is not used:**

Award at most (A0)(A0)(A1)(A1)(ft). Their graph should go through their -8 and 2 for the last (A1)(ft) to be awarded.

[4 marks]

continued...

Question 4 continued

- (f) (i)  $y = -12.5$  OR  $y = 0x - 12.5$  (A1)(A1)

**Note:** Award (A1) for “ $y = \text{constant}$ ”, (A1) for the constant being  $-12.5$ .  
The answer must be an equation.

- (ii) tangent to the graph drawn at  $x = -3$  (A1)(ft)

**Note:** Award (A1) for a horizontal straight-line tangent to curve at approximately  $x = -3$ . Award (A0) if a ruler is not used.  
Follow through from their part (e).

[3 marks]

- (g) decreasing (A1)  
gradient (of tangent line) is negative (at  $x = a$ ) OR  $f'(a) < 0$  (R1)

**Note:** Do not accept “gradient (of tangent line) is  $-6$ ”. Do not award (A1)(R0).

[2 marks]

Total [16 marks]

5. Units are required in part (c)

(a)  $\hat{A}CD = 53^\circ$  (or equivalent) (A1)

**Note:** Award (A1) for  $53^\circ$  (or equivalent) seen.

$$\frac{AD}{\sin 53^\circ} = \frac{70}{\sin 72^\circ} \quad (M1)(A1)$$

**Note:** Award (M1) for substitution into sine rule formula, (A1) for correct substitution.

OR

$$(AD^2 =) 60.2915\dots^2 + 70^2 - 2 \times 70 \times 60.2915\dots \times \cos 53 \quad (A1)(M1)(A1)$$

**Note:** Award (A1) for 53 or 60.2915... seen, (M1) for substitution into cosine rule formula, (A1) for correct substitution.

(AD =) 58.8 (m) (58.7814...) (A1)(G3)

[4 marks]

(b)  $(\cos \hat{A}BC) = \frac{30^2 + 50^2 - 70^2}{2 \times 30 \times 50} \quad (M1)(A1)$

**Note:** Award (M1) for substitution into cosine rule formula, (A1) for correct substitution.

( $\hat{A}BC =$ )  $120^\circ$  (A1)(G2)

[3 marks]

(c)  $A = \frac{1}{2} \times 50 \times 30 \times \sin 120^\circ \quad (M1)(A1)(ft)$

**Note:** Award (M1) for substitution into the area formula, (A1)(ft) for correct substitution.

Award (M0)(A0)(A0) for  $\frac{1}{2} \times 50 \times 30$ .

(A =)  $650 \text{ m}^2$  (649.519...  $\text{m}^2$ ) (A1)(ft)(G2)

**Note:** Follow through from part (b).

[3 marks]

continued...

Question 5 continued

(d) **METHOD 1 (equating part (c) to expression for area of triangle ABC)**

$$649.519\dots = \frac{1}{2} \times 70 \times h \quad (M1)(A1)(ft)$$

**Note:** Award **(M1)** for correctly substituted area of triangle formula. Award **(A1)(ft)** for equating the area formula to their area found in part (c).

$$(h =) 18.6(m) (18.5576\dots) \quad (A1)(ft)$$

**Note:** Follow through from their part (c).

$$20 > 18.5576\dots \quad (R1)(ft)$$

**Note:** Accept “the length of the rope is greater than the altitude of triangle ABC”.

the rope passes inside the triangular plot of land ACD **(A1)(ft)**

**Note:** Follow through from their altitude. The final **(A1)** is contingent on **(R1)** being awarded.

**METHOD 2 (finding  $\hat{CAB}$  or  $\hat{ACB}$  with sine rule and then trig ratio)**

$$\frac{\sin \hat{CAB}}{50} = \frac{\sin 120^\circ}{70} \quad (\hat{CAB} = 38.2132\dots^\circ) \quad (M1)$$

**Note:** Award **(M1)** for their correct substitution into sine rule formula to find  $\hat{CAB}$  or  $\hat{ACB}$ . Follow through from their part (b).

$$(h =) 30 \times \sin (38.2132\dots^\circ) \quad (M1)$$

**Note:** Award **(M1)** for correct substitution of their  $\hat{CAB}$  or  $\hat{ACB}$  into trig formula.

$$(h =) 18.6(m) (18.5576\dots) \quad (A1)(ft)$$

**Note:** Follow through from their part (b).

$$20 > 18.5576\dots \quad (R1)(ft)$$

**Note:** Accept “the length of the rope is greater than the altitude of triangle ABC”.

the rope passes inside the triangular plot of land ACD **(A1)(ft)**

**Note:** Follow through from their altitude. The final **(A1)** is contingent on **(R1)** being awarded.

continued...

Question 5 continued

**METHOD 3 (finding  $\hat{C}AB$  or  $\hat{A}CB$  with cosine rule and then trig ratio)**

$$\cos \hat{A}CB = \frac{50^2 + 70^2 - 30^2}{2(50)(70)} \quad (\hat{A}CB = 21.7867\dots^\circ) \quad \text{(M1)}$$

**Note:** Award **(M1)** for their correct substitution into cosine rule formula to find  $\hat{C}AB$  or  $\hat{A}CB$ .

$$(h =) 50 \times \sin(21.7867\dots^\circ) \quad \text{(M1)}$$

**Note:** Award **(M1)** for correct substitution of their  $\hat{C}AB$  or  $\hat{A}CB$  into trig formula.

$$(h =) 18.6(\text{m}) (18.5576\dots) \quad \text{(A1)(ft)}$$

$$20 > 18.5576\dots \quad \text{(R1)(ft)}$$

**Note:** Accept “the length of the rope is greater than the altitude of triangle ABC”.

the rope passes inside the triangular plot of land ACD (A1)(ft)

**Note:** Follow through from their altitude. The final **(A1)** is contingent on **(R1)** being awarded.

continued...



Question 5 continued

**METHOD 4 (finding area of triangle with height 20, justifying the contradiction)**

$$A = \frac{1}{2}(70)(20) = 700(\text{m}^2) \quad \text{(M1)(A1)}$$

**Note:** Award **(M1)** for correct substitution into area of a triangle formula for a triangle with height 20 and base 70. Award **(A1)** for 700. Award **(M0)(A0)** for unsupported 700 unless subsequent reasoning explains how the 700 was found.

$$700 > 649.519\dots \quad \text{(R1)}$$

if rope exactly touches the AC then this triangle has an area greater than ABC and as the distance AC is fixed the altitude must be less than 20 **(R1)**

**OR**

$$\frac{1}{2}(70)(20) > \frac{1}{2}(70)(\text{height perpendicular to AC}) \text{ and therefore}$$

$$20 > \text{height perpendicular to AC} \quad \text{(R1)(ft)}$$

**Note:** Award **(R1)** for an explanation that recognizes the actual triangle ABC and this new triangle have the same base (70) and hence the height of triangle ABC is less than 20.

therefore, the rope passes inside the triangular plot of land ACD **(A1)(ft)**

**Note:** Other methods, besides those listed here, may be possible. These methods can be summarized in two broad groups: the first is to find the altitude of the triangle, and compare it to 20, and the second is to create an artificial triangle with an altitude of 20 and explain why this triangle is not ABC by relating to area **and** the given lengths of the sides.

**[5 marks]**

**Total [15 marks]**

6. (a)  $(V =) \frac{4\pi r^3}{3} + \pi r^2 l$  (or equivalent) (A1)(A1)

**Note:** Award (A1) for either the volume of a hemisphere formula multiplied by 2 or the volume of a cylinder formula, and (A1) for completely correct expression. Accept equivalent expressions. Award at most (A1)(A0) if  $h$  is used instead of  $l$ .

[2 marks]

(b)  $300 = 4\pi r^2 + 2\pi r l$  (A1)(A1)(A1)

**Note:** Award (A1) for the surface area of a hemisphere multiplied by 2. Award (A1) for the surface area of a cylinder. Award (A1) for the addition of their formulas equated to 300. Award at most (A1)(A1)(A0) if  $h$  is used instead of  $l$ , unless already penalized in part (a).

[3 marks]

(c)  $V = \frac{4\pi r^3}{3} + \pi r^2 \left( \frac{150 - 2\pi r^2}{\pi r} \right)$  (M1)

**Note:** Award (M1) for their correctly substituted formula for  $V$ .

$$V = \frac{4\pi r^3}{3} + 150r - 2\pi r^3$$
 (M1)

**Note:** Award (M1) for correct expansion of brackets and simplification of the cylinder expression in  $V$  leading to the final answer.

$$V = 150r - \frac{2\pi r^3}{3}$$
 (AG)

**Note:** The final line must be seen, with no incorrect working, for the second (M1) to be awarded.

[2 marks]

continued...

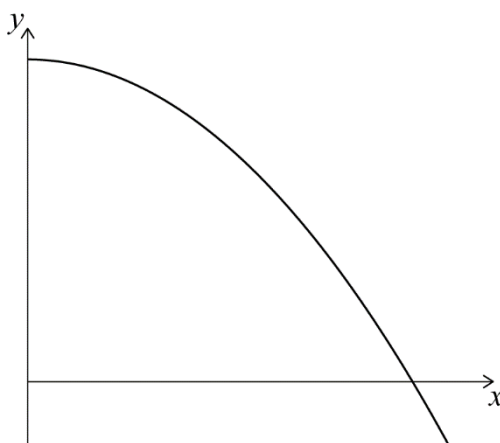
Question 6 continued

(d)  $\left(\frac{dV}{dr} =\right) 150 - 2\pi r^2$  (A1)(A1)

**Note:** Award (A1) for 150. Award (A1) for  $-2\pi r^2$ .  
Award maximum (A1)(A0) if extra terms seen.

[2 marks]

(e)  $150 - 2\pi r^2 = 0$  OR  $\frac{dV}{dr} = 0$  OR sketch of  $\frac{dV}{dr}$  with  $x$ -intercept indicated (M1)



**Note:** Award (M1) for equating their derivative to zero or a sketch of their derivative with  $x$ -intercept indicated.

$r = \sqrt{\frac{150}{2\pi}}$  OR  $r^2 = \frac{150}{2\pi}$  (A1)

$r = \sqrt{\frac{75}{\pi}}$  (AG)

**Note:** The (AG) line must be seen for the preceding (A1) to be awarded.

[2 marks]

continued...

Question 6 continued

$$(f) \quad (l =) \frac{150 - 2\pi \left( \sqrt{\frac{75}{\pi}} \right)^2}{\pi \left( \sqrt{\frac{75}{\pi}} \right)} \quad (M1)$$

**Note:** Award **(M1)** for correct substitution in the given formula for the length of the cylinder.

$$(l =) 0 \text{ (cm)} \quad (A1)(G2)$$

**Note:** Award **(M1)(A1)** for correct substitution of the 3 sf approximation 4.89 leading to a correct answer of zero.

[2 marks]

$$(g) \quad V = 150 \left( \sqrt{\frac{75}{\pi}} \right) - \frac{2\pi \left( \sqrt{\frac{75}{\pi}} \right)^3}{3} \quad \text{OR} \quad V = \frac{4\pi \left( \sqrt{\frac{75}{\pi}} \right)^3}{3} \quad (M1)$$

**Note:** Award **(M1)** for correct substitution in the formula for the volume of the speaker or the volume of a sphere.

$$489 \left( 488.602\dots, 100\sqrt{\frac{75}{\pi}} \right) \text{ (cm}^3\text{)} \quad (A1)(G2)$$

**Note:** Accept 489.795... from use of 3 sf value of  $\sqrt{\frac{75}{\pi}}$ .  
Award **(M1)(A1)(ft)** for correct substitution in their volume of speaker. Follow through from parts (a) and (f).

[2 marks]

$$(h) \quad \text{sphere (spherical)} \quad (A1)(ft)$$

**Note:** Question requires the use of part (f) so if there is no answer to part (f), part (h) is awarded **(A0)**. Follow through from their  $l > 0$ .

[1 mark]

**Total [16 marks]**